Determine the sign of the partial derivative for the function \( f \) whose graph is shown below.

(a) \( f_{xx}(x_0, y_0) \)  
(b) \( f_{yy}(x_0, y_0) \)

Determine the sign of the partial derivative for the function \( f \) whose graph is shown below.

(a) \( f_{xy}(x_0, y_0) \)  
(b) \( f_{xy}(-x_0, y_0) \)

Find the limit, if it exists.

\[
\lim_{(x,y) \to (0,0)} \frac{y^4}{x^4 + 9y^4} \quad \lim_{(x,y) \to (0,0)} \frac{xy \cos y}{9x^2 + y^2} \quad \lim_{(x,y) \to (0,0)} \frac{30x^3y}{5x^4 + y^4} \quad \lim_{(x,y) \to (0,0)} \frac{xy}{\sqrt{x^2 + y^2}}
\]

Use implicit differentiation to find \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \): \( x - z = \arctan(yz) \)

The temperature at a point \((x, y)\) on a flat metal plate is given by \( T(x, y) = \frac{74}{1 + x^2 + y^2} \), where \( T \) is measured in °C and \( x, y \) in meters. Find the rate of change of temperature with respect to distance at the point \((3, 8)\) in the following directions.

(a) the \( x \)-direction  
(b) the \( y \)-direction

A contour map is given for a function \( f \). Use it to estimate \( f_x(2, 1) \) and \( f_y(2, 1) \).
Use the table of values of \( f(x, y) \) to estimate the values of \( f_x(3, 2), f_y(3, 2.2), \) and \( f_{xy}(3, 2). \)

<table>
<thead>
<tr>
<th>( y )</th>
<th>1.8</th>
<th>2.0</th>
<th>2.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>12.5</td>
<td>10.2</td>
<td>9.3</td>
</tr>
<tr>
<td>3.0</td>
<td>18.1</td>
<td>17.5</td>
<td>15.9</td>
</tr>
<tr>
<td>3.5</td>
<td>20.0</td>
<td>22.4</td>
<td>26.1</td>
</tr>
</tbody>
</table>

The paraboloid \( z = 3 - x - x^2 - 2y^2 \) intersects the plane \( x = 4 \) in a parabola. Find parametric equations in terms of \( t \) for the tangent line to this parabola at the point \( (4, 2, -25) \).

The ellipsoid \( 9x^2 + 6y^2 + z^2 = 37 \) intersects the plane \( y = 2 \) in an ellipse. Find parametric equations using the variable \( t \) for the tangent line to this ellipse at the point \( (1, 2, 2) \).

Find an equation of the tangent plane to the given surface at the specified point:
\( z = y \ln(x), (1, 5, 0) \)
\( z = y \cos(x - y), (-7, -7, -7) \)

Find the linear approximation of the function below at the indicated point:
\( f(x, y) = \sqrt{14 - x^2 - 4y^2} \) at \( (1, 1) \)
\( f(x, y) = \ln(x - 5y) \) at \( (6, 1) \)

The wave heights \( h \) in the open sea depend on the speed \( v \) of the wind and the length of time \( t \) that the wind has been blowing at that speed. Values of the function \( h = f(v, t) \) are recorded in feet in the following table.

<table>
<thead>
<tr>
<th>Wind speed (knots)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>5</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>30</td>
<td>9</td>
<td>13</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>40</td>
<td>14</td>
<td>21</td>
<td>25</td>
<td>28</td>
<td>31</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td>50</td>
<td>19</td>
<td>29</td>
<td>36</td>
<td>40</td>
<td>45</td>
<td>48</td>
<td>50</td>
</tr>
<tr>
<td>60</td>
<td>24</td>
<td>37</td>
<td>47</td>
<td>54</td>
<td>62</td>
<td>67</td>
<td>69</td>
</tr>
</tbody>
</table>

Use the table to find a linear approximation to the wave height function when \( v \) is near 40 knots and \( t \) is near 20 hours. (Round the numerical coefficients to two decimal places.)

Estimate the wave heights when the wind has been blowing for 24 hours at 43 knots. (Round the answer to two decimal places.)

Find the differential of each function.
\( z = x^2 \ln(y) \) \(
\) \( v = y \cos(xy) \)
\( w = xy e^{xy} \)

If \( z = 7x^2 + y^2 \) and \( (x, y) \) changes from \( (1, 1) \) to \( (1.05, 1.1) \), compare the values of \( \Delta z \) and \( dz \). (Round your answers to three decimal places.)

The dimensions of a closed rectangular box are measured as 89 cm, 55 cm, and 42 cm, respectively, with a possible error of 0.2 cm in each dimension. Use differentials to estimate the maximum error in calculating the surface area of the box.

Use differentials to estimate the amount of metal in a closed cylindrical can that is 22 cm high and 4 cm in diameter if the metal in the top and the bottom is 0.4 cm thick and the metal in the side is 0.05 cm thick. (Round the answer to two decimal places.)

Let \( W(s, t) = F(u(s, t), v(s, t)) \), where \( F, u, \) and \( v \) are differentiable, and the following applies.
Find \( W_s(1, 5) \) and \( W_t(1, 5) \) given;
\( u(1, 5) = 8 \)
\( v(1, 5) = -3 \)
\( u_t(1, 5) = -3 \)
\( v_t(1, 5) = 8 \)
\( u(1, 5) = -6 \)
\( v_t(1, 5) = -2 \)
\( F_s(8, -3) = 6 \)
\( F_t(8, -3) = -3 \)

Suppose \( f \) is a differentiable function of \( x \) and \( y \), and \( g(u, v) = f(e^u + \sin(v), e^v + \cos(v)) \). Use the table of values to calculate \( g_x(0, 0) \) and \( g_y(0, 0) \).
Use the Chain Rule to find the indicated partial derivatives.

\[ M = xe^{-z^2}, \quad x = 5uv, \quad y = u - v, \quad z = u + v; \quad \frac{\partial M}{\partial u}, \quad \frac{\partial M}{\partial v} \text{ when } u = 1, \quad v = -2 \]

Use the Chain Rule to find the indicated partial derivatives.

\[ u = x^2 + yz, \quad x = pr \cos(\vartheta), \quad y = pr \sin(\vartheta), \quad z = p + r; \quad \frac{\partial u}{\partial p}, \quad \frac{\partial u}{\partial r}, \quad \frac{\partial u}{\partial \vartheta} \text{ when } p = 2, \quad r = 3, \quad \vartheta = 0 \]

Use the following equation to find \( dy/dx \). \( y^2 + x^4 = 2 + ye^{x^2}, \quad \sin(x - y) = xe^y \)

\[ \frac{dy}{dx} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} \]

Use the following equation to find \( \partial z/\partial x \) and \( \partial z/\partial y \). \( xyz = \tan(x + y + z) \quad x - z = \arctan(yz) \quad yz = \ln(x + z) \)

\[ \frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} \]

Find the directional derivative of \( f \) at the given point in the direction indicated by the angle \( \vartheta \).

\[ f(x, y) = ye^x, \quad (0, 1), \quad \vartheta = 4\pi/3 \quad f(x, y) = x \sin(xy), \quad (2, 0), \quad \vartheta = \pi/6 \]

\[ f(x, y) = \sin(2x + 4y), \quad P(4, 2), \quad u = 1/2(\sqrt{3} - j) \]

(a) Find the gradient of \( f \). (b) Evaluate the gradient at the point \( P \).

(c) Find the rate of change of \( f \) at \( P \) in the direction of the vector \( u \).

\[ f(x, y, z) = xe^{2z}, \quad P(1, 0, 3), \quad u = <2/3, \quad -2/3, \quad 1/3> \]

(a) Find the gradient of \( f \).

(b) Evaluate the gradient at the point \( P \). (c) Find the rate of change of \( f \) at \( P \) in the direction of the vector \( u \).

Find the directional derivative of the function at the given point in the direction of vector \( v \).

\[ f(x, y) = 1 + 2xy, \quad (5, 16), \quad v = <8, -6> \]

\[ f(x, y) = \ln(x^2 + y^5), \quad (1, 2), \quad v = <-2, \quad 1> \]

\[ g(r, s) = \arctan(rs), \quad (1, 4), \quad v = 2i + 8 \]

Find the directional derivative of \( f(x, y, z) = xy + yz + zx \) at \( P(6, -6, 7) \) in the direction of \( Q(9, -3, 8) \).

Find the maximum rate of change of \( f \) at the given point and the direction in which it occurs.

\[ f(x, y) = 2y^3/x, \quad (2, 4) \quad f(x, y) = 3 \sin(xy), \quad (0, 5) \]

\[ f(x, y, z) = \sqrt{x^2 + y^2 + z^2}, \quad (-2, 2, 1) \]

Find all the points at which the direction of fastest change of the function \( f(x, y) = x^2 + y^2 - 2x - 6y \) is \( i + j \).

Find equations of the following.

\[ 2(x - 8)^2 + (y - 8)^2 + (z - 2)^2 = 10 \quad (9, 10, 4) \]

\[ x - z = 4 \quad \arctan(yz) \quad (1 + \pi, 1, 1) \]

(a) the tangent plane

(b) the normal line to the given surface at the specified point. (Enter your answer in terms of \( t \).)

If \( f(x, y) = xy \), find the gradient vector \( \nabla f(4, 1) \) and use it to find the tangent line to the level curve \( f(x, y) = 4 \) at the point \( (4, 1) \). Sketch the level curve, the tangent line, and the gradient vector.

If \( g(x, y) = x^2 + y^2 - 4x \), find the gradient vector \( \nabla g(1, 6) \) and use it to find the tangent line to the level curve \( g(x, y) = 33 \) at the point \( (1, 6) \). Sketch the level curve, the tangent line, and the gradient vector.

At what point on the paraboloid \( y = x^2 + z^2 \) is the tangent plane parallel to the plane \( 3x + 2y + 3z = 4 \)?

Are there any points on the hyperboloid \( x^2 - y^2 - z^2 = 3 \) where the tangent plane is parallel to the plane \( z = x + y \)?

Find parametric equations for the tangent line to the curve of intersection of the paraboloid \( z = x^2 + y^2 \) and the ellipsoid \( 3x^2 + y^2 + z^2 = 8 \) at the point \( (-1, 1, 2) \). (Enter your answer in terms of \( t \).)
The plane $y + z = 4$ intersects the cylinder $x^2 + y^2 = 25$ in an ellipse. Find parametric equations for the tangent line to this ellipse at the point $(4, 3, 1)$. (Enter your answer in terms of $t$.)

Find the local maximum and minimum values and saddle point(s) of the function. If you have three dimensional graphing software, graph the function with a domain and viewpoint that reveal all the important aspects of the function. $f(x, y) = x^2 + y^2 - 4xy + 1$ $f(x, y) = x^3 - 3xy + y^3$ $f(x, y) = 8e^{y - x^2}$

Find the absolute maximum and minimum values of $f$ on the set $D$. $f(x, y) = 8 + 4x - 5y$, $D$ is the closed triangular region with vertices $(0, 0), (2, 0), \text{ and } (0, 3)$ $f(x, y) = x^2 + y^2 + 9, D = \{(x, y) \mid x \leq 1, y \leq 1\}$ $f(x, y) = 2x^2 + y^2 + 9, D = \{(x, y) \mid x^2 + y^2 \leq 1\}$

Find the maximum volume of a rectangular box that is inscribed in a sphere of radius $r$.

Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x + 2y + 3z = 6$.

Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(6, 2, 0)$.

Find the points on the surface $y^2 = 9 + xz$ that are closest to the origin.

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