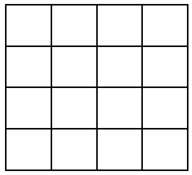
Ch17A Test

Sketch the vector field **F**. **F**$(x, y) = (x - y)\mathbf{i} + x\mathbf{j}$



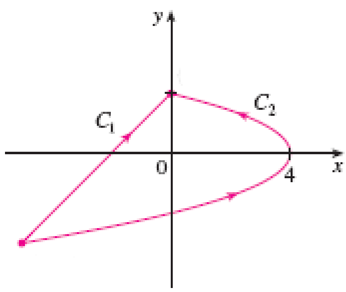
Evaluate the line integral, where C is the given curve. $\int_C xy^4\,ds$. C is the right half of the circle $x^2 + y^2 = 4$ oriented counterclockwise.

Evaluate the line integral $\int_C xyz^2\,ds$. C is the line segment from (-2, 6, 0) to (0, 7, 5).

Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is given by $\mathbf{r}(t)$, $0 \le t \le 1$.
$\mathbf{F}(x, y) = e^{x-1}\mathbf{i} + xy\,\mathbf{j}$, $\mathbf{r}(t) = t^4\mathbf{i} + t^5\mathbf{j}$

Evaluate $\int y^2dx + x\,dy$ along the following paths.

(a) $C = C_1$ is the line segment from (-13, -7) to (0, 6)



(b) $C = C_2$ is the arc of the parabola $x = 36 - y^2$ from (-13, -7) to (0, 6).

Determine whether or not **F** is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$. If it is not, enter NONE.

$\mathbf{F}(x, y) = (e^x\cos(y))\mathbf{i} + (e^x\sin(y))\mathbf{j}$

$\mathbf{F}(x, y) = (\sin(xy) + x\cos(xy)y)\mathbf{i} + (x^2\cos(xy))\mathbf{j}$

Consider **F** and C below.

$\mathbf{F}(x, y, z) = yz\,\mathbf{i} + xz\,\mathbf{j} + (xy + 10z)\mathbf{k}$: C is the line segment from $(2, 0, -1)$ to $(6, 4, 3)$

(a) Find a function f such that $\mathbf{F} = \nabla f$.

(b) Use part (a) to evaluate $\int_C \nabla f \cdot d\mathbf{r}$ along the given curve C.

Find the work done by the force field **F** in moving an object from P to Q.

$\mathbf{F}(x, y) = e^{-y}\mathbf{i} - xe^{-y}\mathbf{j}$, $\quad P(0, 4), Q(4, 0)$

Consider the force field and circle defined below.

$\mathbf{F}(x, y) = x^2\,\mathbf{i} + xy\,\mathbf{j}$, $\quad x^2 + y^2 = 144$

Find the work done by the force field on a particle that moves once around the circle oriented in the clockwise direction.

Find the work done by the force field $\mathbf{F}(x, y) = x\,\mathbf{i} + (y + 7)\mathbf{j}$ in moving an object along an arch of the cycloid $\mathbf{r}(t) = (t - \sin t)\mathbf{i} + (2 - \cos t)\mathbf{j}$, $\quad 0 \le t \le 2\pi$.

Find the work done by the force field $\mathbf{F}(x, y) = x\sin(y)\mathbf{i} + y\,\mathbf{j}$ on a particle that moves along the parabola $y = x^2$ from $(1, 1)$ to $(2, 4)$.

Consider **F** and C below.

$\mathbf{F}(x, y, z) = e^y\mathbf{i} + xe^y\mathbf{j} + (z+1)e^z\mathbf{k}$, $\quad C$: $\mathbf{r}(t) = t\,\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$, $0 \le t \le 1$

(a) Find a function f such that $\mathbf{F} = \nabla f$.

(b) Use part (a) to evaluate $\int_C \nabla f \cdot d\mathbf{r}$ along the given curve C.

Evaluate the line integral by the two following methods.

$\oint (x - y)dx + (x + y)dy$, $\quad C$ is counterclockwise around the circle with center the origin and radius 3.

(a) directly

(b) using Green's Theorem

Evaluate the line integral by the two following methods.

$\oint xy\,dx + x^2y^3\,dy$, $\quad C$ is counterclockwise around the triangle with vertices (0, 0), (1, 0), and (1, 3).

(a) directly
(b) using Green's Theorem

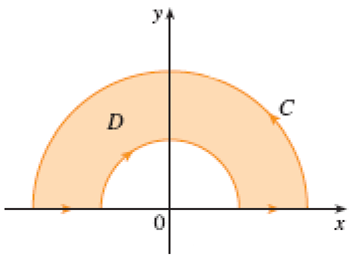
Evaluate the line integral by the two following methods: $\oint x\,dx + y\,dy$, C consists of the line segments from (0, 1) to (0, 0) and from (0, 0) to (1, 0) and the parabola $y = 1 - x^2$ from (1, 0) to (0, 1).
(a) directly
(b) using Green's Theorem

Use Green's Theorem to evaluate the line integral along the given positively oriented curve.
$\int_C xy^2\,dx + 4x^2y\,dy$, C is the triangle with vertices (0, 0), (2, 2), and (2, 4)

Use Green's Theorem to evaluate the line integral along the given positively oriented curve.
$\int_C y^3dx - x^3dy$, C is the circle $x^2 + y^2 = 4$

Use Green's Theorem to evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$. (Check the orientation of the curve before applying the theorem.)
$\mathbf{F}(x, y) = \langle y^2\cos(x), x^2 + 2y\sin(x)\rangle$, C is the triangle from (0, 0) to (1, 3) to (1, 0) to (0, 0).

Evaluate $\oint 2y^2dx + 6xy\,dy$, where C is the boundary of the semiannular region D in the upper half-plane between the circles $x^2 + y^2 = 16$ and $x^2 + y^2 = 25$.



If $\mathbf{F}(x, y) = (-y\mathbf{i} + x\mathbf{j})/(x^2 + y^2)$, show that $\int \mathbf{F}\cdot d\mathbf{r} = 2\pi$ for every positively oriented simple closed path that encloses the origin.