Math 263 Chapter 14 Practice test

Two particles travel along the space curves.
$\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle, \mathbf{r}_2(t) = \langle 1+2t, 1+6t, 1+14t \rangle$
Do the particles collide? Do their paths intersect? Explain.

(a) Sketch the plane curve with the given vector equation. (Do this on paper. Your instructor may ask you to turn in this sketch.) $\mathbf{r}(t) = \langle t-4, t^2+4 \rangle$
(b) Find **r** '(t).
(c) Sketch the position vector **r**(t) and the tangent vector **r** '(t) for $t = -1$. (Do this on paper. Your instructor may ask you to turn in this sketch.)

Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.
$x = 5e^t,\ y = te^{4t},\ z = te^{t^4};\ (5,\ 0,\ 0)$
$x = 4\ln(t),\ y = 8\sqrt{t},\ z = t^5;\ (0,\ 8,\ 1)$

The curves $\mathbf{r}_1 = < 4t, t^2, t^4 >$ and $\mathbf{r}_2 = < \sin(t), \sin(4t), 5t >$ intersect at the origin. Find their angle of intersection, θ correct to the nearest degree.

At what point do the curves $\mathbf{r}_1 = < t, 4 - t, 5 + t^2 >$ and $\mathbf{r}_2 = < 5 - s, s - 1, s^2 >$ intersect?
Find their angle of intersection, θ correct to the nearest degree.

Find the parametric equations for the tangent line to the helix with parametric equations at the point $(0, 3, \pi/2)$. $x = 2\cos(t), y = 3\sin(t), z = t$

Show that if $|\mathbf{r}(t)| = c$ (a constant), then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$ for all t.

Find the length of the curve.
$\mathbf{r}(t) = \langle 2\sin(t), 5t, 2\cos(t) \rangle, -6 \le t \le 6$
$\mathbf{r}(t) = \cos(8t)\,\mathbf{i} + \sin(8t)\,\mathbf{j} + 8\ln\cos(t)\,\mathbf{k}, 0 \le t \le \dfrac{\pi}{4}$
Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$, and the surface $3z = xy$. Find the exact length of C from the origin to the point $(3, 9/2, 9/2)$.

Reparametrize the curve with respect to arc length measured from the point where $t = 0$ in the direction of increasing t. (Enter your answer in terms of s.)
$\mathbf{r}(t) = 3t\mathbf{i} + (3-4t)\mathbf{j} + (3+2t)\mathbf{k}$

Consider the vector function given below. $\mathbf{r}(t) = \langle 3\sin\ t, 8t, 3\cos\ t \rangle$
(a) Find the unit tangent and unit normal vectors **T**(t) and **N**(t).

(b) Find the curvature.

Consider the function.
$y = 9\ln(7x)$
Do the following.
(a) At what point does the curve have maximum curvature?
(b) What happens to the curvature as $x \to \infty$?

Find the vectors **T**, **N**, and **B** at the given point.

$\mathbf{r}(t) = \langle t^2, \frac{2}{3}t^3, t\rangle$, $\langle 9, 18, 3\rangle$.

$\mathbf{r}(t) = \langle \sin(t), \cos(t), \ln(\cos(t))\rangle$, $(0, 1, 0)$.

Find equations of the normal plane and osculating plane of the curve at the given point.

$x = 5\sin(5t)$, $y = 3t$, $z = 5\cos(5t)$; $(0, 3\pi, -5)$

$x = t, y = t^2, z = t^3; (1,1,1)$

At what point on the curve $x = t^3$, $y = 9t$, $z = t^4$ is the normal plane parallel to the plane $6x + 18y - 8z = 3$.

Find the equations of the normal plane and the osculating plane of the helix $\mathbf{r}(t) = 4\cos(t)\mathbf{i} + 4\sin(t)\mathbf{j} + t\mathbf{k}$ at the point $\mathbf{P}(0, 4, \pi/2)$.

Consider the following position function. $r(t) = < 5\cos(t), 6t, 5\sin(t)>$

(a) Find the velocity of a particle with the given position function.

(b) Find the acceleration of a particle with the given position function.

(c) Find the speed of a particle with the given position function.

Find the velocity and position vectors of a particle that has the given acceleration and the given initial velocity and position. $\mathrm{a}(t) = 8\mathbf{i} + 6\mathbf{j}, \mathrm{v}(0) = \mathbf{k}, \mathrm{r}(0) = \mathbf{i}$

A force with magnitude 10 N acts directly upward from the *xy*-plane on an object with mass 2 kg. The object starts at the origin with initial velocity $v(0) = \mathbf{i} - \mathbf{j}$.

(a) Find its position function at time *t*.

(b) Find its speed at time *t*.

A ball is thrown at an angle of 45° to the ground. If the ball lands 92 m away, what was the initial speed of the ball? ($g \approx 9.8$ m/s^2)

A gun is fired with angle of elevation 30°. What is the muzzle speed if the maximum height of the shell is 512 m? (Round the answer to the nearest whole number. $g \approx 9.8$ m/s^2)

A projectile is fired with an initial speed of 450 m/s and angle of elevation 30°. The projectile is fired from a position 190 m above the ground. ($g \approx 9.8$ m/s^2)

(a) Find the range of the projectile. (Round the answer to one decimal place.)

(b) Find the maximum height reached. (Round the answer to one decimal place.)

(c) Find the speed at impact. (Round the answer to the nearest whole number.)

Find the curvature of $r(t) = \langle t, t^2, t^3\rangle$ at the point $(1,1,1)$.

Find the equation of the osculating circles of the ellipse $9x^2 + 4y^2 = 36$ at the points $(2,0)$ and $(0,3)$. Graph both ellipse and osculating circles.